RADIATIVE HEAT TRANSFER BETWEEN SURFACES OF A PLANE ISOTHERMAL LAYER

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The following expression was obtained in [1] for the flux of radiant energy falling on the first surface of a plane isothermal gray layer bounded by gray surfaces:

$$\varepsilon_1^+ = \frac{(1-r_1)\sigma T_1^4 R + \sigma T^4 \varepsilon_{rad} + (1-r_2)\sigma T_2^4 \varepsilon_{tr}}{(1-r_1R)(1-r_2R) - r_1 r_2 \varepsilon_{tr}^2} + \frac{r_2(\varepsilon_{tr} - R)[\sigma T^4 \varepsilon_{rad} + (1-r_1)\sigma T_1^4(\varepsilon_{tr} + R)]}{(1-r_1R)(1-r_2R) - r_1 r_2 \varepsilon_{tr}^2}, \quad (1)$$

where r_1 and r_2 are the reflection coefficients of the first and second surfaces; T_1, T_2, and T are the absolute temperatures of the first surface, second surface, and layer; ε_{rad}, ε_{tr}, and R are, respectively, the emissivity of the layer, the fraction of the total incident radiant energy passing through the layer, and the albedo of the layer. First of all, we note that the results of [1] are still valid if the angular distribution of the true emission of the surfaces and the radiation reflected from the surfaces is the same (not necessarily isotropic). The aim of the present work was to extend (1) to the case of a selective medium and to give a suitable quadrature formula for calculation of the integrals with respect to frequency.

Obviously, if r_1, r_2, and the attenuating characteristics of the medium depend on the frequency, but the hypothesis of identical angular distribution of the true and reflected emission for each frequency is valid, the determination of ε_1^+ requires the following substitutions in (1):

$$\sigma T_1^4 \to B_\omega(T_1)\,d\omega,\quad \sigma T_2^4 \to B_\omega(T_2)\,d\omega,\quad \sigma T^4 \to B_\omega(T)\,d\omega$$

(where $B\omega$ is the Planck distribution function), and integration of the obtained expression with respect to frequency with due regard to the variation of ε_{tr}, ε_{rad}, R, r_1, and r_2 with frequency. This method is more convenient than the method proposed in [2], since the parameters required for calculation of ε_1^+ in [2] depend not only on the characteristics of the layer, but also on r_1 and r_2. In [2] a quadrature formula with weight function e^{-x} was used for integration with respect to frequency. We think that in this case it is more convenient to use the method of quadratures of the highest algebraic accuracy [3] with a Planck weight function.

We consider, for instance, the calculation of the emissivity of an isothermal medium. For the emissivity we have the expression

$$\varepsilon = \frac{1}{\sigma T^4}\int_0^\infty B_\omega(T)\,\varepsilon_{\text{изл}}(\omega)\,d\omega. \quad (2)$$

If we introduce a new variable $y = \omega h/2\pi kT$, then from (2) we obtain

$$\varepsilon = 1 - \frac{1}{3!\,\zeta(4)}\int_0^\infty \frac{y^3}{e^y - 1}\,\varepsilon_{\text{изл}}\left(y\,\frac{2\pi kT}{h}\right)dy, \quad (3)$$

where ζ is the Riemann zeta function. Assuming a weight function $(1/3!\zeta(4))(y^3/(e^y-1))$, the integral of the form $\left(\frac{1}{3!\,\zeta(4)}\right)\int_0^\infty \frac{y^3}{e^y-1} f(y)\,dy$ is represented by the the expression $\sum_{i=1}^{N} a_i f(y_i)$. In the first approximation (one term of the series) we obtain

$$a_1 = 1.000,\quad y_1 = 3.832;$$

in the second approximation

$$a_1 = 0.7215,\quad y_1 = 2.573;$$
$$a_2 = 0.2785,\quad y_2 = 7.095;$$

and in the third approximation

$$a_1 = 0.4864,\quad y_1 = 1.998;$$
$$a_2 = 0.4718,\quad y_2 = 5.105;$$
$$a_3 = 0.0418,\quad y_3 = 10.450.$$

REFERENCES

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